FAST DETECTION OF LO SIGNAL FROM HETERODYNE RECEIVERS

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1. Introduction

Broadcast NTSC and DTV channels are allocated 6 MHz of bandwidth of a predominantly contiguous spectrum in the VHF and UHF bands. A typical "over the air" TV receiver, employing the superheterodyne architecture, emits relatively strong Local Oscillator (LO) signal [1] in channel $n+7$ when tuned to channel n. Fastly detecting the LO signal and thus channel usage could potentially enable spectrally adaptive devices to dynamically utilize vast amounts of idle bandwidth. For example, a potential application of such a detector would be to facilitate local delivery of high-speed wireless media via unused TV bands.

This document considers the detection of LO leakage energy from TV receivers with a DSP capable RF sensor. A frequency uncertainty ϵ is assumed to be uniformly distributed over $[-\delta/2, \delta/2]$. No phase synchronization is assumed. We propose a GLRT criterion, which works with the DFT, for detecting such a tone of unknown phase and frequency offset.

2. System model

Local oscillator leakage from a TV set occurs at 41 MHz above the carrier frequency of the received channel. The LO frequency corresponding to channel n is denoted f_n. Due to variability in RF chipsets, there is an uncertainty ϵ in the exact LO frequency which is modeled as uniform with variance $\delta^2/12$. The goal is to quickly detect a sinusoid of unknown phase and frequency drawn from $\{f_n\}$, where $f_n \sim U[m_n - \delta/2, m_n + \delta/2]$ and $m_n = E[f_n]$.

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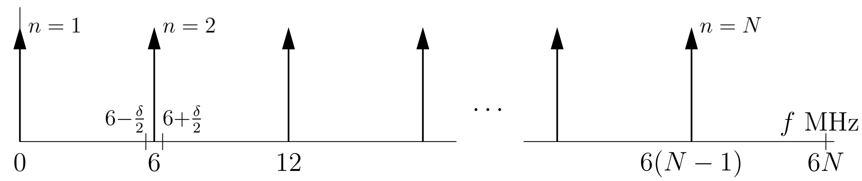
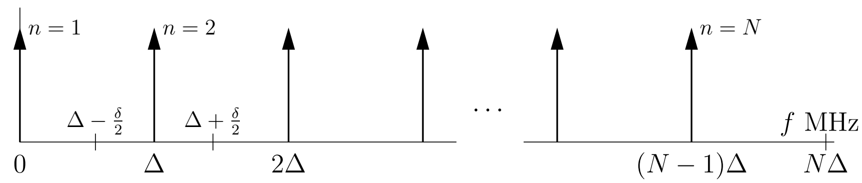


(a) Original spacing of LO frequencies.



(b) Aliased copy for $f_s = 6 - \Delta$.

FIGURE 1. Received signal model.

The channels are assumed to be equispaced by 6 MHz. So that for N channels, the detector is required to monitor a baseband bandwidth of $6N$ MHz. Since typical frequency deviations are on the order of a kHz, only a small fraction of the monitored spectrum yields information relevant to the detector. Given the consistency of channel spacing, aliasing techniques can be employed to losslessly compress the received signal. For example, if $\delta < \Delta < 6/(2N+1)$ MHz, then a sampling rate of $f_s = 6 - \Delta$ MHz will map N "LO channels" in baseband to $[0, N\Delta]$ with a separation of Δ MHz, as depicted in Figure 1. This aliasing technique yields roughly a $2N$-fold reduction in the Nyquest rate. We employ this approach in the following and thus define $m_n = (n-1)\Delta$, for $n = 1, \ldots, N$.

3. Optimal detection

The hypothesis testing problem may be formulated in discrete baseband as follows:

$$\begin{aligned} H_0 &: \; y(l) = w(l), \\ H_n &: \; y(l) = e^{j2\pi f_n T l + j\phi} + w(l), \\ & l = 0, \ldots, K-1, \; n = 1, \ldots, N, \end{aligned}$$

with $T = 1/f_s$. The unknown carrier phase ϕ is uniformly distributed $U[0, 2\pi]$ and $w(l)$ denotes Additive White Gaussian Noise (AWGN) with variance σ^2 per dimension. The notation $\mathbf{y} = [y(0) \cdots y(K-1)]$ denotes a vector of symbols, $\langle \mathbf{y}, \mathbf{x} \rangle = \mathbf{x}^H \mathbf{y}$ is the inner product of vectors $\mathbf{x}$ and $\mathbf{y}$, and K is the DFT length.

Let $\{\mathbf{s}_k\}$, $k = 0, \ldots, K-1$, denote an orthogonal basis of complex exponentials, with $\mathbf{s}_k = [s_k(0) \cdots s_k(K-1)]$, $s_k(l) = \exp(j2\pi kl/K)$. Further define $\mathcal{Q}_n$ as the set of indices $\{k\}$ corresponding to a frequency $q_k = kf_s/K$ in $[m_n - \delta/2, m_n + \delta/2]$, and $\mathcal{Q} = \bigcup_{n=1}^{N} \mathcal{Q}_n$ as the union over all hypotheses. The set $\mathcal{Q}_n$ has the interpretation of a quantizer for the unknown LO frequency under hypothesis n.

The Maximum Likelihood (ML) detector is developed first assuming a discrete unknown LO frequency, taking values in $\mathcal{Q}$, which converges to the continuous case for K large by Riemann approximation. Thus, sufficient statistics for the received symbol sequence are obtained as

$$z(k) = |\langle \mathbf{y}, \mathbf{s}_k \rangle|, k \in \mathcal{Q}. \tag{1}$$

Conditional on LO frequency q_k, the probability density function of $z(k)$ is Ricean, and $z(j)$ is Rayleigh and independent of $z(k)$ for all $j \neq k$ [2]. In particular,

$$P(\mathbf{z}|H_n, q_k) = I_0(\frac{z(k)\sqrt{K}}{\sigma^2}) \exp(\frac{K - \|\mathbf{z}\|^2}{2\sigma^2}) \prod_{j \in \mathcal{Q}} \frac{z(j)}{\sigma^2},$$

where $I_0(x)$ is the zeroth order modified Bessel function of the first kind and $\mathbf{z} = [z(0) \cdots z(K-1)]$.

The Maximum Likelihood (ML) decision rule is given by

$$\hat{n}_{ML} = \arg\max_n \frac{P(\mathbf{z}|H_n)}{P(\mathbf{z}|H_0)} \approx \arg\max_n \frac{1}{\delta TK} \sum_{k \in \mathcal{Q}_n} I_0(\frac{z(k)\sqrt{K}}{\sigma^2}), \tag{2}$$

which converges for K large. In practice, direct evaluation of (2) is cumbersome, and we thus consider an alternative computationally-efficient approach described next.

4. GLRT detection

Due to unknown carrier phase we propose a noncoherent detection algorithm that relies on estimates of the received signal energy in quantized frequency bins. The decision rule is based on the Generalized Likelihood Ratio Test (GLRT), given by

$$\begin{aligned} \hat{n} &= \arg\max_n \max_{k \in \mathcal{Q}_n} \frac{P(\mathbf{z}|H_n, q_k)}{P(\mathbf{z}|H_0)} \\ &= \arg\max_n \max_{k \in \mathcal{Q}_n} z(k), \end{aligned} \tag{3}$$

which may be interpreted as the joint ML estimate of the hypothesis and frequency offset for the received symbols.

When the quantized frequency bins $\mathcal{Q}$ correspond to DFT samples, the above GLRT detector can be implemented directly by summing the appropriate DFT sample over multiple DFT blocks and taking the magnitude. However, that implies the use of a very long DFT in order to achieve sufficient quantization granularity in the regions of frequency uncertainty. In practice, it is more efficient to use a moderate value of K, and to aggregate statistics (over many DFT blocks) from quantizers with a resolution much finer then f_s/K. This is accomplished with a filter matched to the DFT of the leakage signal corresponding to a given LO frequency, as described in the following.

The DFT of the transmitted signal $x(l) = \exp(j2\pi fTl)$ corresponding to LO frequency f is given by

$$X(k) = e^{j\omega_k(K-1)}\frac{\sin(\omega_k K)}{\sin\omega_k},$$

with $\omega_k = \pi(fT - k/K)$, $k = 0, \ldots, K-1$. Thus,

$$Y(k) = e^{j\phi}X(k) + W(k)$$

is the DFT of the first block of received symbols, where $W(k)$ denotes AWGN with variance σ^2 per dimension. As a measure of the relative energy at frequency f, the detector computes the magnitude of the output of a filter matched to $\mathbf{X} = [X(0)\cdots X(K-1)]$, whose input is the DFT of the received symbol sequence. Note that $\mathbf{X}$ reduces to a Kronecker delta when f corresponds to a DFT frequency, say q_k, in which case the magnitude of the matched filter output corresponds to the decision statistic (1).

Since

$$x(l + mK) = e^{j2\pi fTmK}x(l),$$

a frequency dependent phase correction must be applied when combining filter outputs from multiple DFT blocks. Accordingly, the GLRT statistic at frequency f is given by

$$z_f = |\sum_{m=0}^{M-1} e^{-j2\pi fTmK}\langle\mathbf{Y}^{(m)}, \mathbf{X}\rangle|,$$

where $\mathbf{Y}^{(m)}$ denotes the DFT of the mth block of received symbols and M the total number of such blocks.

4.1. **Derotation with the DFT.** Phase correction takes the form of a DFT operation when the frequency quantizer is chosen uniformly according to

$$f = \frac{1}{TK}(k + \frac{q}{M}), \quad k = 0, \ldots, K-1,\ q = 0, \ldots, M-1.$$

The cumulative, phase-compensated symbol sequence for the (k, q)th frequency bin is then given by

$$\mathbf{Y}_q = \sum_{m=0}^{M-1} \mathbf{Y}^{(m)} e^{-j2\pi qm/M},$$

which depends only on the quantization index q. The GLRT criterion is re-stated as

$$\hat{n} = \arg\max_n \max_{(k,q)\in\mathcal{Q}_n} |\langle \mathbf{Y}_q, \mathbf{X}_{kq}\rangle|,$$

with $\mathcal{Q}_n$ defined as the frequency quantizer for hypothesis n and $\mathbf{X}_{kq}$ as the DFT response corresponding to the (k, q)th frequency bin.

5. Results

We first consider the case of known presence of LO leakage signal. The performance of the preceding detector was simulated for $N = 2^4$ channels with a Signal-to-Noise Ratio (SNR) per sample of -20 dB and frequency uncertainty of $\delta = 30$ kHz. The Figure demonstrates that detector performance is dictated by the quantizer resolution, or the product MK. Performance is maximized for values of MK between 2^{13} and 2^{14}. This corresponds to 84 and 168 frequency bins per hypothesis respectively, or a frequency resolution on the order of 100 Hz. In practice, it makes sense to choose $M = K$ in order to dually utilize a common DFT. Since implementing ML detection would require an FFT with length on the order of the values of MK above, the performance with known LO frequencies is provided as a lower bound. Thus, the GLRT detector is within 100 μsec of ML detection time at an error rate of 10^{-3}.

In order to determine if signal is present or not, an energy detector is employed with threshold τ chosen to mitigate falsely detecting the absence of LO signal. Thus LO signal is deemed present if $z_f > \tau$ for some $f \in \mathcal{Q}$. Estimates of the SNR are employed to scale the received signal with respect to the pre-defined threshold.

References

[1] B. Wild and K. Ramchandran. Detecting primary receivers for cognitive radio applications. In *Dynamic Spectrum Access Networks (DySPAN) 2005*, Baltimore, MD, November 2005.

[2] J. Proakis. *Digital Communications*. McGraw-Hill, 1995.

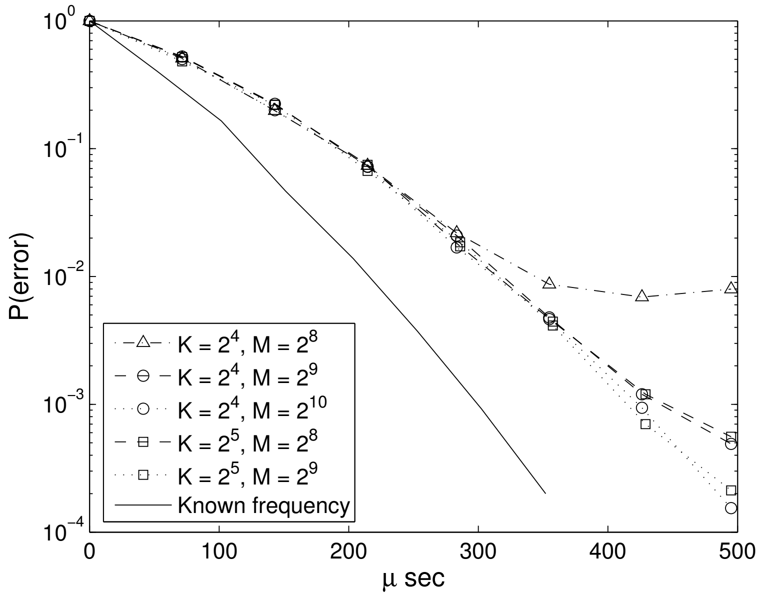


FIGURE 2. GLRT signal detection at -20 dB per sample.