EE201 Linear Systems
HW 10 — Solution

1] Show: $\det\left(\begin{bmatrix} A & B \\ 0 & D \end{bmatrix}\right) = \det(A)\det(D)$

2x2 case: $\det\left(\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\right) = \det(a)\det(d)$

nxn case: Let $\begin{bmatrix} A_{n-1} & B_{n-1} \\ 0 & D_{n-1} \end{bmatrix}$ denote an $(n-1)\times(n-1)$ matrix and suppose

$\det\left(\begin{bmatrix} A_{n-1} & B_{n-1} \\ 0 & D_{n-1} \end{bmatrix}\right) = \det(A_{n-1})\det(D_{n-1}) \quad \forall\ A_{n-1} \in \mathbb{C}^{m\times m},\ B_{n-1} \in \mathbb{C}^{m\times p},$

and $D_{n-1} \in \mathbb{C}^{p\times p}$, where $n-1 = m+p$.

Let $\begin{bmatrix} A_n & B_n \\ 0 & D_n \end{bmatrix}$ denote an $n\times n$ matrix, where $A_n \in \mathbb{C}^{(m+1)\times(m+1)}$,

$B_n \in \mathbb{C}^{(m+1)\times p}$, $D_n \in \mathbb{C}^{p\times p}$ and $n = m+p+1$.

Then: $$\det\left(\begin{bmatrix} A_n & B_n \\ 0 & D_n \end{bmatrix}\right) = \sum_{k=1}^{m+1} [A_n]_{k,1} (-1)^{k-1} A_n\begin{pmatrix} 1 \cdots k-1 & k+1 \cdots m+1 \\ 2 \cdots k & k+1 \cdots m+1 \end{pmatrix} \det(D_n)$$

$$= \det(A_n)\det(D_n).$$

2] $\dot{z} = Az + bu, \quad y = c^T z, \quad z(0) = 0.$

$$Z_q^{(k)}(s) = \left[\frac{\gamma_{l_k}^{(k)}}{(s-\lambda_k)^{l_k-q+1}} + \frac{\gamma_{l_k-1}^{(k)}}{(s-\lambda_k)^{l_k-q}} + \cdots + \frac{\gamma_q^{(k)}}{s-\lambda_k} \right] \cdot U(s)$$

$$\Rightarrow z_q^{(k)}(t) = \left[\frac{\gamma_{l_k}^{(k)} t^{l_k-q} e^{\lambda_k t}}{(l_k-q)!} + \frac{\gamma_{l_k-1}^{(k)} t^{l_k-q-1} e^{\lambda_k t}}{(l_k-q-1)!} + \cdots + \gamma_q^{(k)} e^{\lambda_k t} \right] * u(t)$$

$$\Rightarrow h(t) = \sum_{k=1}^{p} \sum_{q=1}^{l_k} \delta_q^{(k)} z_q^{(k)}(t) = \sum_{k=1}^{p} e^{\lambda_k t} \sum_{q=1}^{l_k} \delta_q^{(k)} \sum_{m=0}^{l_k-q} \frac{\gamma_{q+m}^{(k)} t^m}{m!}$$

$$= \sum_{k=1}^{p} e^{\lambda_k t} \sum_{m=0}^{l_k-1} \frac{t^m}{m!} \sum_{q=1}^{l_k-m} \delta_q^{(k)} \gamma_{q+m}^{(k)}$$

$\Rightarrow$ $H(s)$ is irreducible iff $\delta_1^{(k)} \neq 0$ and $\gamma_{l_k}^{(k)} \neq 0$, $k = 1, \dots, p$.
(i.e. $\{A, b, c\}$ is contr. + obs.)