E6201 Linear Systems
Homework 10 (*due:* Apr. 12)

1. Show $\det\left(\begin{bmatrix} A & B \\ 0 & D \end{bmatrix}\right) = \det(A)\det(D)$.

2. Let

$$A = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_p \end{bmatrix}, \quad b = \begin{bmatrix} \gamma_1^{(1)} \\ \gamma_2^{(1)} \\ \vdots \\ \gamma_{l_p}^{(p)} \end{bmatrix}, \quad c = \begin{bmatrix} \delta_1^{(1)} \\ \delta_2^{(1)} \\ \vdots \\ \delta_{l_p}^{(p)} \end{bmatrix},$$

where

$$J_k = \begin{bmatrix} \lambda_k & 1 & & \\ & \lambda_k & 1 & \\ & & \ddots & 1 \\ & & & \lambda_k \end{bmatrix}.$$

Show the impulse response of the system $\{A, b, c\}$ is equal to:

$$h(t) = \sum_{k=1}^{p} e^{\lambda_k t} \sum_{m=0}^{l_k - 1} \frac{t^m}{m!} \sum_{q=1}^{l_k - m} \delta_q^{(k)} \gamma_{q+m}^{(k)}$$

Specify a condition on $\{\delta_q^{(k)}\}$ and $\{\gamma_m^{(k)}\}$ for system to be both controllable and observable.