E6201 Linear Systems
Homework 9 (*due:* Apr. 5)

1. Let $\{A, b, c\}$ denote an nth order realization of an LTI dynamical system such that $\text{rank}\big(\mathcal{O}(c, A)\big) = r < n$. Show there exists a nonsingular matrix T such that:
   a. $\bar{A} = T^{-1}AT$, $\bar{b} = T^{-1}b$, and $\bar{c}^T = c^T T$, where
   $$\bar{A} = \begin{bmatrix} \bar{A}_o & 0 \\ \bar{A}_{\bar{o}o} & \bar{A}_{\bar{o}} \end{bmatrix}, \qquad \bar{b} = \begin{bmatrix} \bar{b}_o \\ \bar{b}_{\bar{o}} \end{bmatrix}, \qquad \text{and } \bar{c} = \begin{bmatrix} \bar{c}_o \\ 0 \end{bmatrix},$$
   b. $\{\bar{A}_o, \bar{b}_o, \bar{c}_o\}$ is an order r observable realization, and
   c. $c^T(sI - A)^{-1}b = \bar{c}_o{}^T(sI - \bar{A}_o)^{-1}\bar{b}_o$.
2. Let $\{A, b, c\}$ denote a minimal realization of an LTI system. Show there exists a nonsingular symmetric matrix T (i.e. $T = T^T$) such that $A^T = T^{-1}AT$ and $b = Tc$.
   [*Hint:* See Example 2.4-6 in Kailath.]
3. Problem 2.4-13 in Kailath. [*Hint:* See section 2.4.3, p. 138.]