E6201 Linear Systems
Homework 6 (*due:* Mar. 1)

1. Let $x(t)$ denote an infinitely differentiable function at zero. Show the inverse Laplace transform of
$$\sum_{k=0}^{\infty} x^{(k)}(0)\, s^{-k-1}$$
is equal to $x(t)$ in a neighborhood of zero.
2. Let $y^{(3)} + a_1 y^{(2)} + a_2 y^{(1)} + a_3 y = b_1 u^{(2)} + b_2 u^{(1)} + b_3 u$. Assume zero initial conditions and show the transfer function is given by
$$H(s) = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}.$$
3. Let $\dot{x} = Ax + bu$ and $y = c^T x$. Show
$$H(s) = \frac{c^T \mathrm{Adj}(sI - A) b}{\det(sI - A)}.$$
4. Let $\dot{x} = A_c x + b_c u$ and $y = {c_c}^T x$ denote a 3rd order system in controller form, where
$$A_c = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ b_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, c_c = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$
Show
$$H(s) = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}.$$
[*Hint:* The result for an $n$th order system follows from part 1 of A.33 (Appendix 9).]
5. Questions 2.2-22 and 2.2-23 in Kailath.