E6201 Linear Systems
Homework 5 (*due:* Feb. 23)

1. Suppose $x(t)$ is twice differentiable and that $\mathcal{L}[\dot{x}(t)]$ and $\mathcal{L}[\ddot{x}(t)]$ exist. Show that $\mathcal{L}[\ddot{x}(t)] = s^2\mathcal{L}[x(t)] - sx(0) - \dot{x}(0)$.
2. Let $f(t)$ denote a continuous function in a neighborhood of zero and let
$$p_\Delta(t) = \begin{cases} 1/\Delta, & 0 \le t < \Delta \\ 0, & else \end{cases}.$$
Show
$$f(0) \triangleq \int \delta(t)f(t)dt = \lim_{\Delta\to 0}\int p_\Delta(t)f(t)dt.$$
Hint: Apply the mean value theorem to evaluate the integral $\int p_\Delta(t)f(t)dt$ for each Δ sufficiently small and find the limit as Δ approaches zero.
3. Let $J_A = T^{-1}AT$ denote the Jordan normal form corresponding to the matrix A. Suppose A^{-1} exists and find an expression for it.
4. Let A_c denote a companion matrix in so-called controller form, i.e.
$$A_c = \begin{bmatrix} -a_1 & -a_2 & & -a_{n-1} & -a_n \\ 1 & 0 & & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ & & \ddots & & \\ 0 & 0 & & 1 & 0 \end{bmatrix}.$$
and suppose $a(\lambda) = 0$, where $\lambda \in \mathbb{C}$ and $a(s) = s^n + a_1 s^{n-1} + \cdots + a_n$.
Show that $[\lambda^{n-1}\ \lambda^{n-2}\ \cdots\ 1]^T$ is a characteristic vector of A_c.

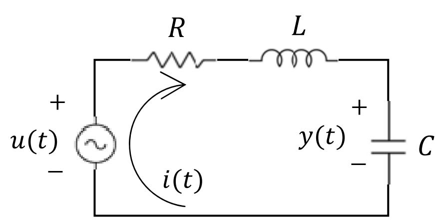

Fig. 1. An RLC circuit.

5. Let $R = 2\Omega$, $L = 1\text{H}$, and $C = 0.5\text{F}$ in Fig. 1.
   a. Assume the input voltage $u(t) = 0$ for all t, $y(0) = 1$ and $i(0) = 0$. Solve for the output voltage $y(t)$.
   b. Let $u(t) = (1/2)\exp(-at)$ for $t \ge 0, a > 0$. Assume the initial conditions are the same as part (a.), i.e. $y(0) = 1$ and $i(0) = 0$, and solve for the output voltage $y(t)$.

   Hint: Kirchoff's voltage law for this circuit is given by:
$$u(t) - R\,i(t) - L\frac{d}{dt}i(t) - y(t) = 0,$$
   where the output voltage is $y(t) = C^{-1}\int_0^t i(\tau)d\tau + 1$.