E6201 Linear Systems
Homework 2 (*due:* Feb. 2)

1. Show that the number of linearly independent columns of a matrix is equal to the number of linearly independent rows. Thus, $\mathrm{rank}(A) = \mathrm{rank}(A^T)$.
2. Let A denote an $n \times n$ matrix. Show $\det(sI - A)$ is an nth order polynomial in s.
3. *Cayley-Hamilton Theorem.* Let $a(s) = \det(sI - A)$ denote the characteristic equation of the $n \times n$ matrix A. Assume A has n linearly independent characteristic vectors and let $a(s) = s^n + a_1 s^{n-1} + \cdots + a_n$. Show:

$$a(A) = A^n + a_1 A^{n-1} + \cdots + a_n I = 0.$$

Possible approach: Let x denote an arbitrary vector in $\mathbb{R}^n$. Show $a(A)x = 0$.