E6201 Linear Systems
HW1 Solutions – Spring 2012

<u>1)</u> $p \in \Pi(1,\dots,n)$

Let q_p denote the inverse permutation of p.

Then $q_p \in \Pi(1,\dots,n)$, $q_p(p(k)) = k$, $k = 1,\dots,n$, and $\mu(p) = \mu(q_p)$.

$\left[p_1 \neq p_2 \Leftrightarrow q_{p_1} \neq q_{p_2}\right] \Rightarrow \{q_p : p \in \Pi(1,\dots,n)\} = \Pi(1,\dots,n)$

Hence,
$$\det(A) = \sum_{p\in\Pi(1,\dots,n)} (-1)^{\mu(p)} \prod_{k=1}^{n} [A]_{k,p(k)}$$
$$= \sum_{p\in\Pi(1,\dots,n)} (-1)^{\mu(p)} \prod_{k=1}^{n} [A]_{q_p(p(k)),\,p(k)}$$
$$= \sum_{q\in\Pi(1,\dots,n)} (-1)^{\mu(q)} \prod_{k'=1}^{n} [A]_{q(k'),\,k'}$$

<u>2)</u> <u>a)</u> Show $\det(A^T) = \det(A)$.

By HW1, Q1 :
$$\det(A^T) = \sum_{p\in\Pi(1,\dots,n)} (-1)^{\mu(p)} \prod_{k=1}^{n} [A^T]_{k,p(k)}$$
$$= \sum_{p\in\Pi(1,\dots,n)} (-1)^{\mu(p)} \prod_{k=1}^{n} [A^T]_{p(k),k}$$
$$= \sum_{p\in\Pi(1,\dots,n)} (-1)^{\mu(p)} \prod_{k=1}^{n} [A]_{k,p(k)}$$
$$= \det(A) .$$

<u>b)</u> $AA^{-1} = I \Rightarrow \det(A)\det(A^{-1}) = \det(I) \Rightarrow \det(A^{-1}) = \det(A)^{-1}$

<u>3)</u> $[L]_{ij} = 0 ,\ j > i \Rightarrow$

$$\det(L) = [L]_{11}\, C_{11}(L) = [L]_{11}[L]_{22}\, C_{11}(L_1) = \cdots = \prod_{j=1}^{n} [L]_{jj}$$

(L_1 equals L with 1st column and 1st row deleted)